Model Answer
Set- III
Std. – 10th EM/Semi
Subject – Geometry

Time : 2 Hrs. **Marks : 40**

Q.1.A Choose the correct alternative.

i) b) parallel to each other

ii) b) 25 cm

iii) a) Equilateral triangle

iv) b) (3, 4, 5)

Q.1.B Solve the following questions.

i) $PR \times PS = PQ^2$ ……[Tangent secant segments theorem]

$\therefore 8 \times PS = 12^2$

$\therefore 8 \times PS = 144$

$\therefore PS = \frac{144}{8}$

$\therefore PS = 18$ units

ii) In ΔABC, $DE \parallel BC$ …….[Given]

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ……[Basic Proportionality theorem]

$\therefore \frac{3}{4} = \frac{6}{EC}$

$\therefore EC = \frac{6 \times 4}{3} = 8$ cm

iii) Diagonal of a square $= \sqrt{2} \times$ side

$= \sqrt{2} \times 10$

$= 10\sqrt{2}$ cm

iv) $1\ m^3 = 1000$ litres

$\therefore$ Capacity of reservoir $= 86.24 \times 1000 = 86240$ litres

Q.2.A Complete the following activities. (Any two)

i) line $l \parallel$ X-axis

Slope of line $l = \frac{3-3}{2-1} =$ [0]

$\therefore$ Slope of any line parallel to X-axis is [Zero]

line n $\parallel$ Y-axis

Slope of line n $= \frac{3-2}{-1-(-1)} = \frac{1}{0} =$ [Not defined]

$\therefore$ Slope of any line parallel to Y-axis [cannot be defined]

ii) **Ans:** In ΔABC and ΔDEF,

$\angle A \cong \angle D$

$\angle B \cong \angle E$ } ... [**Each angle is of measure 60°**]

$\therefore \Delta ABC \sim \Delta DEF$ ...[AA test of similarity]

$\therefore$ [$\frac{A(\Delta ABC)}{A(\Delta DEF)}$] = [$\frac{AB^2}{DE^2}$] ...[Theorem of areas of similar triangles]

$\therefore \frac{1}{2} = \frac{4^2}{DE^2}$

$\therefore DE^2 = 4^2 \times 2$

$\therefore DE =$ [$4\sqrt{2}$ **units**] ...[Taking square root of both sides]

iii) $AB = BC$ ...[Given]

$\therefore \angle BAC = \angle BCA$ ...[Isosceles triangle theorem]

$\therefore \angle BAC = \boxed{45°}$

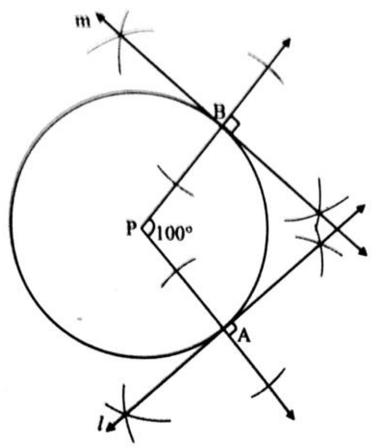
$\therefore AB = BC = \boxed{\frac{1}{\sqrt{2}}} \times AC$ ...[Side opposite to 45°]

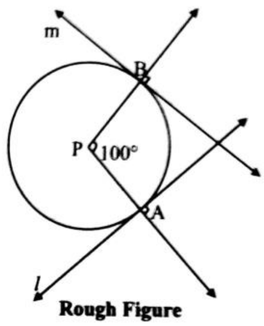
$= \boxed{\frac{1}{\sqrt{2}}} \times \sqrt{8} = \frac{1}{\sqrt{2}} \times 2\sqrt{2}$

$\therefore AB = BC = \boxed{2 \text{ units}}$

Q.2.B Solve the following questions. (Any four)

i)





Rough Figure

ii) Given: For the cuboid,

Length (l) = 44 cm, breadth (b) = 21 cm,

height (h) = 12 cm

For the cone, height (H) = 24 cm

To find: Radius of base of the cone (R).

Volume of cuboid = volume of cone

$\therefore l \times b \times h = \frac{1}{3} \pi R^2 H$

$\therefore 44 \times 21 \times 12 = \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24$

$\therefore R^2 = \frac{44 \times 21 \times 12 \times 3 \times 7}{22 \times 24}$

$\therefore R^2 = 21 \times 21$

$\therefore R = 21$ cm ....[Taking square root of both sides]

$\therefore$ The radius of the base of the cone is 21 cm.

iii) □ MRPN is a cyclic quadrilateral. ....[Given]

$\therefore \angle R + \angle N = 180°$ ......[Theorem of cyclic quadrilateral]

$\therefore 5x - 13 + 4x + 4 = 180$

$\therefore 9x - 9 = 180$

$\therefore 9x = 189$

$\therefore x = \frac{189}{9}$

$\therefore x = 21$

$\therefore \angle R = 5x - 13$

$= 5 \times 21 - 13$

$= 105 - 13 = 92°$

$\angle N = 4x + 4$

$= 4 \times 21 + 4$

$= 84 + 4 = 88°$

$\therefore m\angle R = 92°$ and $m\angle N = 88°$

iv) In ΔPQR, point S is the midpoint of side QR. ....[Given]

$\therefore PQ^2 + PR^2 = 2\,PS^2 + 2\,SR^2$ ....[Apollonius theorem]

$\therefore 11^2 + 17^2 = 2\,(13)^2 + 2\,SR^2$

$\therefore 121 + 289 = 2(169) + 2\,SR^2$

$\therefore 410 = 338 + 2\,SR^2$

$\therefore 2\,SR^2 = 410 - 338$

$\therefore 2\ SR^2 = 72$

$\therefore SR^2 = \frac{72}{2} = 36$

$\therefore SR = \sqrt{36}$ ....[Taking square root of both sides]

$= 6$ units

Now, QR = 2 SR ....[S is the midpoint of QR]

$= 2 \times 6$

$\therefore$ QR = 12 units

v) Here, $(x_1, y_1) = (22, 20)$,

$(x_2, y_2) = (0, 16)$

Let the co-ordinates of the midpoint be (x, y).

$\therefore$ By midpoint formula,

$x = \frac{x_1 + x_2}{2} = \frac{22 + 0}{2} = 11$

$y = \frac{y_1 + y_2}{2} = \frac{20 + 16}{2} = \frac{36}{2} = 18$

$\therefore$ The co-ordinates of the midpoint of the segment joining (22, 20) and (0, 16) are (11, 18).

Q.3.A Complete the following activities. (Any one)

i)

We know that, slope of line = $\boxed{\frac{y_2 - y_1}{x_2 - x_1}}$

Slope of side AB $= \frac{2-(-7)}{-1-(-4)} = \frac{2+7}{-1+4} = \frac{9}{3} = 3$

Slope of side BC $= \frac{5-2}{8-(-1)} = \frac{3}{8+1} = \frac{3}{9} = \boxed{\frac{1}{3}}$

Slope of side CD $= \frac{-4-5}{5-8}$

$= \frac{-9}{-3} = \boxed{3}$

Slope of side AD $= \frac{-4-(-7)}{5-(-4)} = \frac{-4+7}{5+4} = \frac{3}{9} = \frac{1}{3}$

$\therefore$ Slope of side AB = Slope of side $\boxed{CD}$

$\therefore$ side AB || side CD

Slope of side $\boxed{BC}$ = Slope of side AD

$\therefore$ side BC || side AD

Both the pairs of opposite sides of □ABCD are parallel.

$\therefore$ The quadrilateral formed by joining the points A, B, C and D is a $\boxed{\text{parallelogram}}$.

ii)

By theorem of touching circles, points X, Z, Y are collinear.

$\therefore$ ∠XZA ≅ $\boxed{∠BZY}$ ...[Vertically opposite angles]

Let ∠XZA = ∠BZY = a ...(i)

Now, seg XA ≅ seg XZ ... $\boxed{\text{Radii of the same circle}}$

$\therefore$ ∠XAZ ≅ $\boxed{∠XZA}$ = a ...(ii)[Isosceles triangle theorem]

Similarly, seg YB ≅ seg YZ ... $\boxed{\text{Radii of the same circle}}$

$\therefore$ ∠BZY = ∠ZBY = a ...(iii) $\boxed{\text{Isosceles triangle theorem}}$

$\therefore$ ∠XAZ = ∠ZBY ...[From (i), (ii) and (iii)]

$\therefore$ radius XA || radius YB ... $\boxed{\text{Alternate angles test}}$

Q.3.B Solve the following questions. (Any two)

i) Given: For the cylindrical roller, diameter (d) = 120 cm, height (h) = 84 cm,
Number of rotations required to level the ground = 200
Rate of levelling = Rs. 10 per m^2
To find: Expenditure of levelling the ground.
Diameter of roller (d) = 120 cm

$\therefore$ Radius of roller (r) = $\frac{d}{2} = \frac{120}{2}$ = 60 cm

$\therefore$ Curved surface area of roller = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 60 \times 84$$
$$= 2 \times 22 \times 60 \times 12$$
$$= 31680 \text{ cm}^2$$
$$= \frac{31680}{100 \times 100} \text{ m}^2 \quad ....[\because 1 \text{ m} = 100 \text{ cm}]$$
$$= 3.168 \text{ m}^2$$

Now, area of ground levelled in one rotation = 3.168 m^2

$\therefore$ Area of ground levelled in 200 rotations = 3.168 × 200
= 633.6 m^2

Rate of levelling = Rs. 10 per m^2

$\therefore$ Expenditure of levelling = 633.6 × 10
= Rs. 6336

$\therefore$ The expenditure of levelling the ground is Rs. 6336.

ii)

$\Delta ABC \sim \Delta LMN$ ...[Given]

$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{LN}$ ...(i) [Corresponding sides of similar triangles]

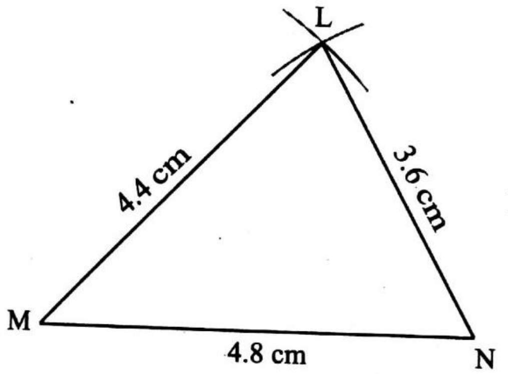
But, $\frac{BC}{MN} = \frac{5}{4}$ ...(ii)[Given]

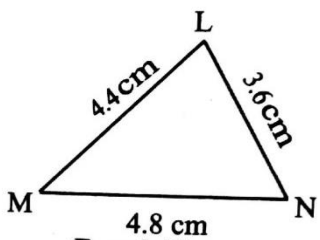
$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{LN} = \frac{5}{4}$ ...[From (i) and (ii)]

$\therefore \frac{5.5}{LM} = \frac{6}{MN} = \frac{4.5}{LN} = \frac{5}{4}$

$\therefore \frac{5.5}{LM} = \frac{5}{4}$ | Also, $\frac{6}{MN} = \frac{5}{4}$ | and, $\frac{4.5}{LN} = \frac{5}{4}$

$\therefore LM = \frac{5.5 \times 4}{5}$ = 4.4 cm | $\therefore MN = \frac{6 \times 4}{5}$ = 4.8 cm | $\therefore LN = \frac{4.5 \times 4}{5}$ = 3.6 cm





Rough Figure

iii) Let AB and CD represent the heights of the two buildings, and BD represent the width of the road.

AB = 12 m

BD = 15 m

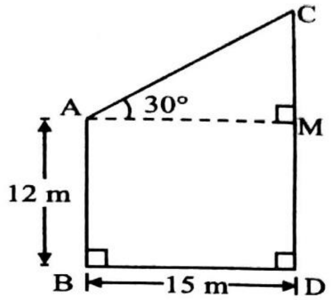
Draw seg AM ⊥ seg CD.

Angle of elevation = ∠CAM = 30°

In □ABDM,

∠B = ∠D = 90°

∠M = 90° ...[seg AM ⊥ seg CD]



∴ ∠A = 90° ...[Remaining angle of □ABDM]

∴ □ABDM is a rectangle. ...[Each angle is 90°]

∴ AM = BD = 15 m, DM = AB = 12 m ...[Opposite sides of a rectangle]

In right angled ΔAMC,

$$\tan 30° = \frac{CM}{AM}$$

∴ $\frac{1}{\sqrt{3}} = \frac{CM}{15}$

∴ $CM = \frac{15}{\sqrt{3}}$

∴ $CM = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ...[On rationalising the denominator]

∴ $CM = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$ m

Now, CD = DM + CM ...[C–M–D]

$= (12 + 5\sqrt{3})$ m

∴ **The height of the second building is $(12 + 5\sqrt{3})$ m.**

iv) seg AC and seg BD are the radii of circles with centres A and B respectively and line CD is the common tangent to those circles. ...[Given]

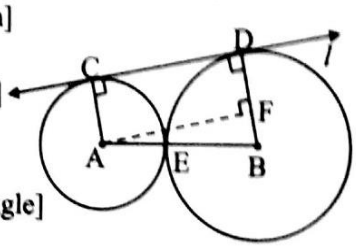
∴ ∠ACD = ∠BDC = 90° ...[Tangent theorem]

∠AFD = 90° ...[Construction]

∴ ∠CAF = 90° ...[Remaining angle of □AFDC]

∴ □AFDC is a rectangle.

... (i) [Each angle is of measure 90°]



∴ AC = DF = 4 cm ...[Opposite sides of a rectangle]

Now, BD = BF + DF ...[B – F – C]

∴ 6 = BF + 4

∴ BF = 2 cm

Also, AB = AE + EB ...[The distance between the centres of circles touching externally is equal to the sum of their radii]

= 4 + 6 = 10 cm

Now, in ΔAFB, ∠AFB = 90°...[Construction]

∴ $AB^2 = AF^2 + BF^2$ ...[Pythagoras theorem]

∴ $10^2 = AF^2 + 2^2$

∴ $100 = AF^2 + 4$

∴ $AF^2 = 96$

∴ $AF = \sqrt{96}$

$= \sqrt{16 \times 6}$ ...[Taking square root of both sides]

$= 4\sqrt{6}$ cm

But, CD = AF ...[Opposite sides of a rectangle]

∴ $CD = 4\sqrt{6}$ cm

Q.4 Solve the following questions. (Any two)

i) Radius of spherical ball (r) = 3 cm

Volume of one sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times (3)^3$

$= \frac{4}{3} \times \pi \times 27 = 36\pi$

∴ Volume of 14 spheres = $14 \times 36\pi = 504\pi$

For cylindrical jar,

radius (R) = 10 cm, height (H) = 15 cm

Volume of water in the jar = $\pi R^2 H = \pi \times (10)^2 \times 15 = 1500\pi$

Total volume of water + Volume of 14 spheres

$= 1500\pi + 504\pi = 2004\pi$

Let the new height of water be h

Volume of water in the cylinder when spherical balls are immersed = 2004π

$\therefore \pi r^2 h = 2004\pi$

$\therefore h = \frac{2004}{r^2} = \frac{2004}{(10)^2}$

$\therefore h = \frac{2004}{100} = 20.04$ cm

∴ New level upto which water is filled in the jar is 20.04 cm.

ii) L.H.S. $= (1 + \tan\theta)^2 + (1 + \cot\theta)^2$

$= 1 + 2\tan\theta + \tan^2\theta + 1 + 2\cot\theta + \cot^2\theta$

...$[(a+b)^2 = a^2 + 2ab + b^2]$

$= (1 + \tan^2\theta) + 2\tan\theta + 2\cot\theta + (1 + \cot^2\theta)$

$= \sec^2\theta + 2(\tan\theta + \cot\theta) + \operatorname{cosec}^2\theta$

...$[\because 1 + \tan^2\theta = \sec^2\theta,\ 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$

$= \sec^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta . \cos\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{1}{\sin\theta . \cos\theta}\right) + \operatorname{cosec}^2\theta$ ...$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sec^2\theta + 2\left(\frac{1}{\sin\theta} \times \frac{1}{\cos\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2(\operatorname{cosec}\theta \cdot \sec\theta) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\sec\theta \cdot \operatorname{cosec}\theta + \operatorname{cosec}^2\theta$

$= (\sec\theta + \operatorname{cosec}\theta)^2$ = R.H.S.

$\therefore$ $\mathbf{(1 + \tan\theta)^2 + (1 + \cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2}$

iii) **Given:** LM = LN, ∠PMN = ∠NMQ

To prove: $\frac{LM^2}{LQ^2} = \frac{LP}{LQ}$.

Proof: In ΔLMN,

	$LM = LN$	...[Given]
$\therefore$	$\angle LMN \cong \angle LNM$	...(i)[Isosceles triangle theorem]
	$\angle LPM = \angle PNM + \angle PMN$	...[Remote interior angle theorem]
	$\angle LPM = \angle LNM + \angle NMQ$	...[L–P–N and $\angle PMN = \angle NMQ$]
$\therefore$	$\angle LPM = \angle LMN + \angle NMQ$	...[From (i)]
	$\angle LPM = \angle LMQ$	...(ii)[Angle addition property]
	In ΔLPM and ΔLMQ,	
	$\angle LPM \cong \angle LMQ$	...[From (ii)]
	$\angle PLM \cong \angle MLQ$	...[Common angle]
$\therefore$	$\Delta LPM \sim \Delta LMQ$	...[by AA test of similarity]
	$\frac{A(\Delta LPM)}{A(\Delta LMQ)} = \frac{LM^2}{LQ^2}$	...(iii)[Theorem of areas of similar triangles]
	But, $\frac{A(\Delta LPM)}{A(\Delta LMQ)} = \frac{LP}{LQ}$	...(iv)[Triangles having equal heights]
$\therefore$	$\frac{LM^2}{LQ^2} = \frac{LP}{LQ}$	...[From (iii) and (iv)]

Q.5 Solve the following questions. (Any one)

i) ***Proof:***

Let 'O' be the centre of the circle.

In □OPCR,

	$\angle P = \angle R = 90°$	...[Tangent is perpendicular to the radius]
	$\angle C = 90°$	...[Given]
$\therefore$	$\angle O = 90°$	...[Remaining angle of □OPCR]
$\therefore$	□OPCR is a rectangle.	...[By definition]
	Also, $OP = OR = r$	...[Radii of the same circle]
$\therefore$	□OPCR is a square.	...[A rectangle is a square if adjacent sides are congruent]
$\therefore$	$CP = CR = OP = OR = r$	...(i)
	$AP = AQ = x$	...(ii) [Lengths of tangent segments drawn from an external point to a circle are equal]
	$BQ = BR = y$	...(iii)
	$BC = CR + BR$	...[C–R–B]
$\therefore$	$a = r + y$	...(iv) [From (i), (iii) and given]
	$AC = CP + AP$	...[A–P–C]
$\therefore$	$b = r + x$	...(v) [From (i), (ii) and given]
	$AB = AQ + BQ$	...[A–Q–B]
$\therefore$	$c = x + y$	...(vi) [From (ii), (iii) and given]

Consider,

$a + b - c = (r + y) + (r + x) - (x + y)$ ...[From (iv), (v) and (vi)]

$\therefore$ $a + b - c = r + y + r + x - x - y$

$\therefore$ $a + b - c = 2r$

$\therefore$ $\mathbf{2r = a + b - c}$

ii)

Proof: □DEFG is a square. ...[Given]

∴ seg GF ∥ seg DE ...[Sides of a square]

i.e. seg GF ∥ seg BC ...[B–D–E–C]

seg GF ∥ seg BC and seg AB is their transversal.

∴ $\angle FGA \cong \angle ABC$ ...[Corresponding angles]

∴ $\angle FGA \cong \angle GBD$ ...(i)[A–G–B, B–D–C]

seg GF ∥ seg BC and seg AC is their transversal.

∴ $\angle GFA \cong \angle ACB$ ...[Corresponding angles]

∴ $\angle GFA \cong \angle FCE$ ...(ii)[A–F–C, C–E–B]

In ΔFGA and ΔGBD,

$\angle FAG \cong \angle GDB$ ...[Each angle is of measure 90°]

$\angle FGA \cong \angle GBD$ ...[From (i)]

∴ $\Delta FGA \sim \Delta GBD$ ...(iii)[AA test of similarity]

In ΔFGA and ΔCFE,

$\angle FAG \cong \angle CEF$ ...[Each angle is of measure 90°]

$\angle GFA \cong \angle FCE$ ...[From (ii)]

∴ $\Delta FGA \sim \Delta CFE$ ...(iv)[AA test of similarity]

∴ $\Delta GBD \sim \Delta CFE$ ...[From (iii) and (iv)]

∴ $\frac{DG}{EC} = \frac{BD}{FE}$ ...[Corresponding sides of similar triangles]

∴ $DG \times FE = BD \times EC$ ...(v)

But, $DG = FE = DE$ ...(vi)[Sides of a square]

∴ $DE \times DE = BD \times EC$ ...[From (v) and (vi)]

∴ $\mathbf{DE^2 = BD \times EC}$